DETERMINATION OF COMPLEX DIELECTRIC AND MAGNETIC PROPERTIES OF MATERIALS USING A LEAST SQUARE FIT METHOD BASED ON VON HIPPEL'S TECHNIQUE

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Abstract

The paper describes a modified Von Hippel technique for determination of complex dielectric and magnetic properties of materials. The technique uses least square fit to derive the scatter matrix parameters for calibrating out internal reflection between the sample and the reflection measuring equipment. Least square fit is also used to determine best values of μ and ε.

Introduction

The standard Von Hippel's technique to determine dielectric and magnetic properties of materials consists of placing a sample of material in a transmission line usually coax or a waveguide backed by a short and a shorted stub usually quarter wave long and noting the change in the complex reflection coefficient between the empty line and the line containing the sample.

A considerable improvement in the accuracy of this technique can be obtained if two simple procedures are added. The first procedure calibrates out internal reflections due to discontinuities within the measurement system. The second procedure involves the termination of the sample with a number of different stubs or cavities to provide redundance in measurements. Both of these procedures are carried out in identical fashion and results are evaluated by a least square fit.

System Calibration

Figure 1 illustrates the experimental setup. The basic components are: reflection measuring equipment such as a network analyzer or a slotted line, a connecting network, a sample holder and a terminating shorted stub or a cavity.

The essential concept involved in the system calibration is to determine the scatter matrix parameters of the connecting network which can be used to relate the measured reflection coefficient to the true reflection coefficient.

The calibrating procedure consists of measuring the reflection coefficients of an empty sample holder by terminating it with a number of different shorted stubs or cavities ranging in depth from zero (short) to half a wavelength. To determine the scatter matrix of the connecting network, a minimum of three measurements are required and at least four or five cavities should be used to provide the redundancy for the least square fit.

The scatter matrix is represented by

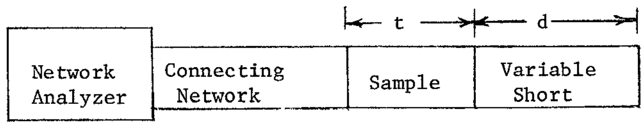
$$\begin{pmatrix} V_1' \\ V_2' \end{pmatrix} = \begin{pmatrix} r_1 & T \\ T & r_2 \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \end{pmatrix}$$

where the various parameters are explained in Figure 2.

Each i^{th} measurement leads to a linear equation in the form of

$$\rho_{2i}\, r_1 + \rho_{1i}\, r_2 + r_3 = a_i \qquad (1)$$

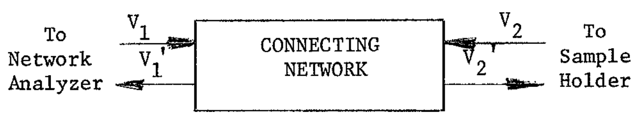
where r_1, r_2 and r_3 are the three unknowns to be determined by the least square fit.



Coax or Waveguide

MEASUREMENT SETUP

Figure 1



$\rho_1 = V_1'/V_1$ = measured reflection coefficient

$\rho_2 = V_2'/V_2$ = 1/(true reflection coefficient)

SCATTER MATRIX REPRESENTATION OF THE CONNECTING NETWORK

Figure 2

The above quantities are defined as follows

ρ_{1i} = complex reflection coefficient measured with the i^{th} cavity

ρ_{2i} = $-\exp(2j\beta(d_i+t))$

β = 2π/wavelength in the empty line or waveguide

d_i = length of the terminating i^{th} shorting stub or cavity

t = thickness of the sample

r_3 = $r_1 r_2 + T^2$

a_i = $\rho_{1i}\rho_{2i}$

Note that Eq. (1) is linear in r_1, r_2, and r_3, and the procedure for the least square can be found in any standard textbook on engineering mathematics. The only caution that must be exercised is to note that these equations are complex. The error residue S is given by

$$S = \sum_i X_i X_i^* \qquad (2)$$

*denotes conjugate

where $X_i = \rho_{2i} r_i + \rho_{1i}\, r_2 + r_3 - a_i$

Partial differentiation of S with respect to the conjugates of the three unknowns gives rise to three simultaneous linear equations in r_1, r_2 and r_3 which are the least square values and which can be readily solved for the best values of the scatter matrix.

If the optimum values are substituted in the original equation, S/n where n is the number of measurements, is a measure of self-consistency of the measurements. A typical value of S/n with five cavities should be less than 10^{-3}, if measurements are carried out with a network analyzer where reflection coefficient is determined to within $\pm$ .1 dB and phase to within $\pm$ .2°.

Measurements of Complex μ and ε

Sample measurements are carried out in exactly the same way as the calibration measurements except that the sample holder now contains the sample.

Since μ and ε are to be determined a minimum of two measurements are required, but again the use of four or five cavities will provide redundancy for the least square fit.

By using standard transmission line equations, it can be shown that each i^{th} measurement may be expressed as

$$A_i\sigma_i + (A_i - \sigma_i)\, X + Y = 0 \qquad (3)$$

where A_i = $(1-\rho_o/(1+\rho_o))$

ρ_o = true reflection coefficient (including scatter matrix correction)

σ_i = $\tan(j\phi_i)$

ϕ_i = electrical length of the i^{th} stub or cavity

X = T/η

Y = $T\eta$

T = $\tanh(-\gamma t)$

t = sample thickness

γ = complex propagation constant of the tranmission line containing the sample.

η = complex characteristic impedance of the transmission line containing the sample divided by the characteristic impedance of the empty line.

Equation 3 is linear in X and Y and the procedures for finding the least square values of X and Y is identical to that used to determine the scatter matrix parameters.

To relate γ and η to μ and ε the following relationships are used.

In the case of a coaxial air line (4)

$\mu\varepsilon$ = $\gamma^2/\beta_o{}^2$

μ/ε = η^2

β_o = 2π/wavelength in air

In the case of a waveguide TE_{10} mode

$\mu\varepsilon$ = $(\beta_c{}^2-\gamma^2)/\beta_o{}^2$ (5)

μ = $\eta\gamma/j\beta_g$

β_c = 2π/cutoff wavelength

= π/waveguide width

β_g = 2π/wavelength in the empty waveguide

$\beta_o{}^2$ = $\beta_g{}^2 + \beta_c{}^2$

Note that equation (5) reduces to equation (4) if $\beta_c = 0$.

Table I illustrates a typical measurement and compares the results computed with and without least square fit. The measurement was made in a coaxial line at 3 GHz and involved a Teflon sample .256" thick. It will be noted that the least square results are considerably better than the results which were obtained by averaging six non-redundant measurements as evidenced by the value of μ which should be 1+j0. The very small residues in the least square data provide a very high confidence factor that the redundant measurements are self-consistent and the results are accurate within a fraction of a percent.

The table also illustrates a fundamental difficulty with this technique in measuring losses in a material with loss tangents of less than 1%. Very low losses are best measured in resonant cavities by changes in the loaded and unloaded Q.

TABLE I

ILLUSTRATION OF A TYPICAL MEASUREMENT

Sample: Teflon .256" thick λ = 10.00 cm

RAW DATA

Cavity No	Cavity In	Calibration dB	Calibration Deg.	Sample dB	Sample Deg.
1	.000	18.00	180.0	18.00	177.1
2	.102	18.00	160.4	18.02	152.9
3	1.575	17.85	-108.2	17.87	-115.1
4	2.559	17.85	70.1	17.73	28.2

NON-REDUNDANT RESULTS

Calibration Utilized Cavities 1, 3 and 4

Cavity Combination	Mu Real	Mu Imag	Epselon Real	Epselon Imag
1-2	1.065	-.019	1.600	.338
1-3	1.061	-.015	1.948	.033
1-4	1.059	-.015	2.133	-.005
2-3	1.033	.009	2.038	-.045
2-4	1.025	.001	2.142	-.011
3-4	.998	-.002	2.149	-.008
Average	1.040	-.007	2.002	.050
St. Dev.	.026	.011	.212	.143

LEAST SQUARE FIT RESULTS

	Mu Real	Mu Imag	Epselon Real	Epselon Imag
	1.001	-.001	1.999	.003

Residues: Calib. .00007 Sample .00003

Conclusions

The introduction of the scatter matrix compensates for internal reflections caused by such items as waveguide junctions, waveguide to coax adaptors and coax connectors between the network analyzer and the sample. Multiplicity of measurements increases accuracy and minimizes experimental errors. Accuracies in excess of 99.5% can be readily achieved with routine measurements.

References

1. Arthur R. Von Hippel, Dielectric Materials and Applications, John Wiley & Sons, 1954, pp 63-87 and pp 119-121.